SPP seminar

- DT theory for minimal surfaces in hyperbolic space -

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1 Abstract

In a 1996 paper, Donaldson and Thomas [DT96] suggested that it may be possible to find topological invariants of a Calabi-Yau three-fold by applying the machinary of Morse-Floer theory.
Consider the twistor space Z of hyperbolic 4-space (denoted $\mathcal{H}^4$ with projection $\pi : Z \to \mathcal{H}^4$). This is a 6-dimensional manifold whose structure is close to one of a true Calabi-Yau, in particular, its the tangent space TZ has the pointwise algebraic structure of $\mathbb{C}^3$. Analogous to the Donaldson and Thomas situation, there is a functional $\mathcal{F}$ on surfaces (of Z) whose critical points are the complex curves.

The first step in building a coherent Floer theory is to understand the (anti-)gradient flow lines of $\mathcal{F}$ that are asymptotic to complex curves i.e. paths of surfaces $\{\Sigma_t\}_{t\in\mathbb{R}}$ satisfying $\frac{\partial}{\partial t}\Sigma_t = -\text{grad}\mathcal{F}$ and $\Sigma_{\pm\infty}$ are complex curves.
There is a way to interpret those objects geometrically. In fact, the map that associates a complex curve $\Sigma \subset Z$ to its projection $\pi(\Sigma) \subset \mathcal{H}^4$ is a bijection between (non-vertical) complex curves and minimal surfaces of $\mathcal{H}^4$. On the other hand, the gradient flows $\{\Sigma_t\}$, seen as 3-manifolds $A^3 = \{(z \in \Sigma_t, t)\}$ inside of $Z \times \mathbb{R}$, have the property that their tangent space have the algebraic structure of the imaginary quaternions $\operatorname{Im}\mathbb{H} \cong T_aA$ $(a \in A)$ sitting inside the imaginary octonions $\operatorname{Im}\mathbb{O} \cong T_a(Z \times \mathbb{R})$. Such spaces are called associatives submanifolds. The aim of the PhD is to get a better understanding of such associatives A that are asymptotic to complex curves $\Sigma_{\pm\infty}$.

References

[DT96] S. K. Donaldson and R. P. Thomas. Gauge theory in higher dimensions. In *Conference on Geometric Issues in Foundations of Science in honor of Sir Roger Penrose's 65th Birthday*, pages 31–47, 6 1996.