Partial modules and comodules

William Hautekiet
Joint work with Eliezer Batista et Joost Vercruysse

15 december 2021
SPP Seminar

Partial actions

Does SO(3) act on the Atomium ?

Partial actions

A *partial action* of a group G on a set X is a collection $(D_g, \alpha_g)_{g \in G}$ where $D_g \subseteq X, \alpha_g : D_{g^{-1}} \to D_g$ are bijections such that

Partial actions

A *partial action* of a group G on a set X is a collection $(D_g, \alpha_g)_{g \in G}$ where $D_g \subseteq X, \alpha_g : D_{g^{-1}} \to D_g$ are bijections such that

- $D_e = X$ and $\alpha_e = X$;
- $\alpha_g(D_{g^{-1}} \cap D_h) = D_g \cap D_{gh}$ and $\alpha_g \circ \alpha_h = \alpha_{gh}$ on $D_{h^{-1}} \cap D_{(gh)^{-1}}$.

Partial actions

A *partial action* of a group G on a set X is a collection $(D_g, \alpha_g)_{g \in G}$ where $D_g \subseteq X, \alpha_g : D_{g^{-1}} \to D_g$ are bijections such that

- $D_e = X$ and $\alpha_e = X$;
- $\alpha_g(D_{g^{-1}} \cap D_h) = D_g \cap D_{gh}$ and $\alpha_g \circ \alpha_h = \alpha_{gh}$ on $D_{h^{-1}} \cap D_{(gh)^{-1}}$.

Example

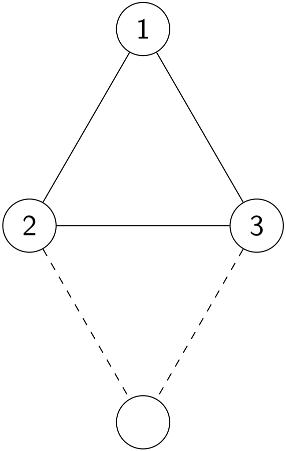
Let $\alpha : G \to \mathrm{Sym}(Y)$ be an action on a set Y and take $X \subseteq Y$. Then putting

$$D_g = X \cap g(X), \qquad \alpha_g = \alpha(g)_{|D_{g^{-1}}}$$

for every $g \in G$ defines a partial action of G on X.

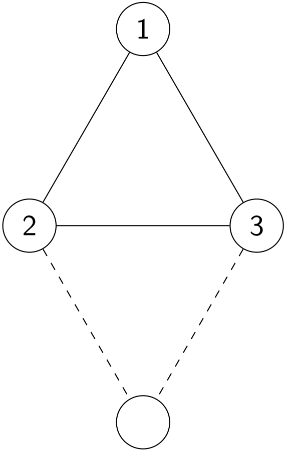
Example

$C_2 \times C_2$ acts partially on an equilateral triangle.



Example

$C_2 \times C_2$ acts partially on an equilateral triangle.



$$e : \begin{cases} 1 \mapsto 1 \\ 2 \mapsto 2 \\ 3 \mapsto 3 \end{cases}$$

$$a : \begin{cases} 1 \mapsto 1 \\ 2 \mapsto 3 \\ 3 \mapsto 2 \end{cases}$$

$$b : \begin{cases} 2 \mapsto 2 \\ 3 \mapsto 3 \end{cases}$$

$$c : \begin{cases} 2 \mapsto 3 \\ 3 \mapsto 2 \end{cases}$$

Partial modules

Idea : linearisation of partial actions. Let k be a field.

$$\pi : kG \otimes kX \to kX : g \otimes x \mapsto g \cdot x = \begin{cases} \alpha_g(x) & \text{if } x \in D_{g^{-1}}, \\ 0 & \text{else.} \end{cases}$$

Partial modules

Idea : linearisation of partial actions. Let k be a field.

$$\pi : kG \otimes kX \to kX : g \otimes x \mapsto g \cdot x = \begin{cases} \alpha_g(x) & \text{if } x \in D_{g^{-1}}, \\ 0 & \text{else.} \end{cases}$$

Remark : in general,

$$g \cdot (h \cdot x) \neq (gh) \cdot x$$

but we have

$$g \cdot (h \cdot (h^{-1} \cdot x)) = (gh) \cdot (h^{-1} \cdot x).$$

Partial modules

Idea : linearisation of partial actions. Let k be a field.

$$\pi : kG \otimes kX \to kX : g \otimes x \mapsto g \cdot x = \begin{cases} \alpha_g(x) & \text{if } x \in D_{g^{-1}}, \\ 0 & \text{else.} \end{cases}$$

Remark : in general,

$$g \cdot (h \cdot x) \neq (gh) \cdot x$$

but we have

$$g \cdot (h \cdot (h^{-1} \cdot x)) = (gh) \cdot (h^{-1} \cdot x).$$

Moreover,

$$\pi(g^{-1})\pi(g)$$

is the projection on the subspace generated by $D_{g^{-1}}$, these define idempotents in $\mathrm{End}(kX)$.

Partial modules

Definition

A partial module over kG is a vector space M equipped with a linear map

$$\pi : kG \otimes M \to M$$

such that for all $g, h \in G, m \in M$

- $e \cdot m = m$;
- $g \cdot (h \cdot (h^{-1} \cdot m)) = (gh) \cdot (h^{-1} \cdot m)$;
- $g^{-1} \cdot (g \cdot (h \cdot m)) = g^{-1} \cdot ((gh) \cdot m)$

Partial modules

Definition

A partial module over kG is a vector space M equipped with a linear map

$$\pi : kG \otimes M \to M$$

such that for all $g, h \in G, m \in M$

- $e \cdot m = m$;
- $g \cdot (h \cdot (h^{-1} \cdot m)) = (gh) \cdot (h^{-1} \cdot m)$;
- $g^{-1} \cdot (g \cdot (h \cdot m)) = g^{-1} \cdot ((gh) \cdot m)$

Does the category of partial modules ${}_{kG}\mathrm{PMod}$ contain more information about G than ${}_{kG}\mathrm{Mod}$?

Partial modules

Definition

A partial module over kG is a vector space M equipped with a linear map

$$\pi : kG \otimes M \to M$$

such that for all $g, h \in G, m \in M$

- $e \cdot m = m$;
- $g \cdot (h \cdot (h^{-1} \cdot m)) = (gh) \cdot (h^{-1} \cdot m)$;
- $g^{-1} \cdot (g \cdot (h \cdot m)) = g^{-1} \cdot ((gh) \cdot m)$

Does the category of partial modules ${}_{kG}\mathrm{PMod}$ contain more information about G than ${}_{kG}\mathrm{Mod}$?
E. g. $C_2 \times C_2$ has 11 irreducible one-dimensional partial modules and 1 three-dimensional partial module.

Partial modules over Hopf algebras

Group algebra kG

- Multiplication :

$$kG \otimes kG \to kG : g \otimes h \mapsto gh,$$

- Unit : $k \to kG : a \mapsto ae$,

Hopf algebra H

- Multiplication :

$$H \otimes H \to H$$

- Unit : $k \to H : a \mapsto a1_H$

Partial modules over Hopf algebras

Group algebra kG

- Multiplication :

$$kG \otimes kG \to kG : g \otimes h \mapsto gh,$$

- Unit : $k \to kG : a \mapsto ae$,
- Comultiplication Δ :

$$kG \to kG \otimes kG : g \mapsto g \otimes g,$$

- Counit $\epsilon : kG \to k : g \mapsto 1$,
- Inverses : $kG \to kG : g \mapsto g^{-1}$.

Hopf algebra H

- Multiplication :

$$H \otimes H \to H$$

- Unit : $k \to H : a \mapsto a1_H$

Partial modules over Hopf algebras

Group algebra kG

- Multiplication :

$$kG \otimes kG \to kG : g \otimes h \mapsto gh,$$

- Unit : $k \to kG : a \mapsto ae$,
- Comultiplication Δ :

$$kG \to kG \otimes kG : g \mapsto g \otimes g,$$

- Counit $\epsilon : kG \to k : g \mapsto 1$,
- Inverses : $kG \to kG : g \mapsto g^{-1}$.

Hopf algebra H

- Multiplication :

$$H \otimes H \to H$$

- Unit : $k \to H : a \mapsto a1_H$
- Comultiplication Δ :

$$H \to H \otimes H : h \mapsto h_{(1)} \otimes h_{(2)},$$

- Counit $\epsilon : H \to k$
- Antipode $S : H \to H$.

Partial modules over Hopf algebras

Group algebra kG

- Multiplication :

$$kG \otimes kG \to kG : g \otimes h \mapsto gh,$$

- Unit : $k \to kG : a \mapsto ae$,
- Comultiplication Δ :

$$kG \to kG \otimes kG : g \mapsto g \otimes g,$$

- Counit $\epsilon : kG \to k : g \mapsto 1$,
- Inverses : $kG \to kG : g \mapsto g^{-1}$.

Hopf algebra H

- Multiplication :

$$H \otimes H \to H$$

- Unit : $k \to H : a \mapsto a1_H$
- Comultiplication Δ :

$$H \to H \otimes H : h \mapsto h_{(1)} \otimes h_{(2)},$$

- Counit $\epsilon : H \to k$
- Antipode $S : H \to H$.

Partial kG-module :

- $e \cdot m = m$,
- $g \cdot (h \cdot (h^{-1} \cdot m)) = (gh) \cdot (h^{-1} \cdot m)$.

Partial modules over Hopf algebras

Group algebra kG

- Multiplication :

$$kG \otimes kG \to kG : g \otimes h \mapsto gh,$$

- Unit : $k \to kG : a \mapsto ae,$
- Comultiplication Δ :

$$kG \to kG \otimes kG : g \mapsto g \otimes g,$$

- Counit $\epsilon : kG \to k : g \mapsto 1,$
- Inverses : $kG \to kG : g \mapsto g^{-1}.$

Partial kG-module :

- $e \cdot m = m,$
- $g \cdot (h \cdot (h^{-1} \cdot m)) = (gh) \cdot (h^{-1} \cdot m).$

Hopf algebra H

- Multiplication :

$$H \otimes H \to H$$

- Unit : $k \to H : a \mapsto a1_H$
- Comultiplication Δ :

$$H \to H \otimes H : h \mapsto h_{(1)} \otimes h_{(2)},$$

- Counit $\epsilon : H \to k$
- Antipode $S : H \to H.$

Partial H-module :

- $1_H \cdot m = m,$
- $h \cdot (h'_{(1)} \cdot (S(h'_{(2)}) \cdot m)) = (hh'_{(1)}) \cdot (S(h'_{(2)}) \cdot m).$

Partial modules over Hopf algebras

Theorem (M. Alves, E. Batista, J. Vercruysse)

If the antipode is invertible, then the category of partial modules over H is equivalent to the category of modules over the "partial Hopf algebra" H_{par} :

$${}_H\mathrm{PMod} \simeq {}_{H_{par}}\mathrm{Mod}.$$

Partial modules over Hopf algebras

Theorem (M. Alves, E. Batista, J. Vercruysse)

If the antipode is invertible, then the category of partial modules over H is equivalent to the category of modules over the "partial Hopf algebra" H_{par} :

$$ {}_{H}\mathrm{PMod} \simeq {}_{H_{par}}\mathrm{Mod}. $$

H_{par} is the quotient of the free algebra generated by the symbols $[h]$ for $h \in H$ by the relations

$$ \begin{aligned} [1_H] &= 1_{H_{par}} \\ [h][h'_{(1)}][S(h'_{(2)})] &= [hh'_{(1)}][S(h'_{(2)})] \\ [h_{(1)}][S(h_{(2)})][h'] &= [h_{(1)}][S(h_{(2)})h'] \end{aligned} $$

Partial modules over Hopf algebras

Let A be the subalgebra generated by

$$\{[h_{(1)}][S(h_{(2)})] \mid h \in H\}.$$

Then H is a *Hopf algebroid* over A and there is a strong monoidal and closed forgetful functor

$$_{H_{par}}\mathrm{Mod} \to {}_A\mathrm{Mod}_A.$$

Partial modules over Hopf algebras

Let A be the subalgebra generated by

$$\{[h_{(1)}][S(h_{(2)})] \mid h \in H\}.$$

Then H is a *Hopf algebroid* over A and there is a strong monoidal and closed forgetful functor

$${}_{H_{par}}\mathrm{Mod} \to {}_A\mathrm{Mod}_A.$$

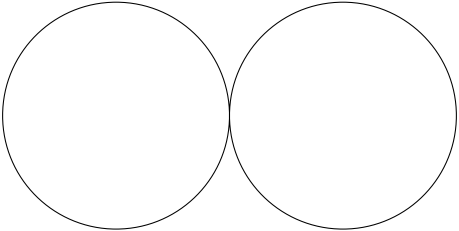
Remark

The forgetful functor ${}_H\mathrm{PMod} \to \mathrm{Vect}_k$ has a left adjoint

$$\mathrm{Vect}_k \to {}_{H_{par}}\mathrm{Mod} \simeq {}_H\mathrm{PMod} : V \mapsto H_{par} \otimes V$$

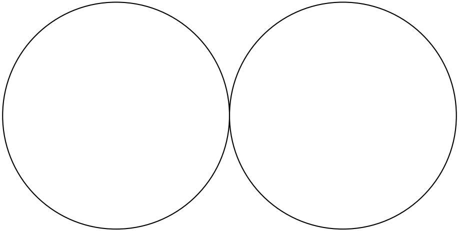
$${}_{H_{par}}\mathrm{Hom}(H_{par} \otimes V, M) \cong \mathrm{Hom}_k(V, M)$$

Partial comodules



$G = \mathrm{SO}(2) \times C_2$

Partial comodules



$$G = \mathrm{SO}(2) \times C_2$$

A linear map

$$\pi : kG \otimes V \to V$$

induces a linear map

$$\rho : V \to V \otimes \mathcal{O}(G).$$

Partial comodules

Definition

A partial comodule over H is a vector space M equipped with a linear map $\rho : M \to M \otimes H$ such that

- $M \xrightarrow{\rho} M \otimes H \xrightarrow{M \otimes \epsilon} M$ is the identity on M;

Partial comodules

Definition

A partial comodule over H is a vector space M equipped with a linear map $\rho : M \to M \otimes H$ such that

- $M \xrightarrow{\rho} M \otimes H \xrightarrow{M\otimes\epsilon} M$ is the identity on M;
- $M \xrightarrow{\rho} M \otimes H \xrightarrow{\rho\otimes H} M \otimes H \otimes H \xrightarrow[M\otimes\Delta\otimes H]{\rho\otimes H^2} M \otimes H \otimes H \otimes H \xrightarrow{M\otimes H^2\otimes S} M \otimes H \otimes H \otimes H \xrightarrow{M\otimes H\otimes\mu} M \otimes H \otimes H$;

Partial comodules

Definition

A partial comodule over H is a vector space M equipped with a linear map $\rho : M \to M \otimes H$ such that

- $M \xrightarrow{\rho} M \otimes H \xrightarrow{M\otimes\epsilon} M$ is the identity on M;
- $M \xrightarrow{\rho} M \otimes H \xrightarrow{\rho\otimes H} M \otimes H \otimes H \underset{M\otimes\Delta\otimes H}{\overset{\rho\otimes H^2}{\longrightarrow}} M \otimes H \otimes H \otimes H \xrightarrow{M\otimes H^2\otimes S} M \otimes H \otimes H \otimes H \xrightarrow{M\otimes H\otimes\mu} M \otimes H \otimes H$;
- $M \xrightarrow{\rho} M \otimes H \xrightarrow{\rho\otimes H} M \otimes H \otimes H \underset{M\otimes H\otimes\Delta}{\overset{\rho\otimes H^2}{\longrightarrow}} M \otimes H \otimes H \otimes H \xrightarrow{M\otimes H\otimes S\otimes H} M \otimes H \otimes H \otimes H \xrightarrow{M\otimes\mu\otimes H} M \otimes H \otimes H$.

Is $\mathrm{PMod}^H \simeq \mathrm{Mod}^C$ for some coalgebra C?

Example

Let H_4 be the Sweedler Hopf algebra, $H_4 = \langle 1, g, x, y \rangle$. Let $M = k[z]$ equipped with

$$\rho : k[z] \to k[z] \otimes H_4 : z^n \mapsto z^n \otimes \frac{1+g}{2} + z^{n+1} \otimes y.$$

Then $(k[z], \rho)$ is a partial comodule over H_4.

Example

Let H_4 be the Sweedler Hopf algebra, $H_4 = \langle 1, g, x, y \rangle$. Let $M = k[z]$ equipped with

$$\rho : k[z] \to k[z] \otimes H_4 : z^n \mapsto z^n \otimes \frac{1+g}{2} + z^{n+1} \otimes y.$$

Then $(k[z], \rho)$ is a partial comodule over H_4.
However, M contains and infinite descending chain of partial subcomodules

$$k[z] \supset k[z]^{\geq 1} \supset k[z]^{\geq 2} \supset \cdots \supset k[z]^{\geq n} \supset \cdots$$

Example

Let H_4 be the Sweedler Hopf algebra, $H_4 = \langle 1, g, x, y \rangle$. Let $M = k[z]$ equipped with

$$\rho : k[z] \to k[z] \otimes H_4 : z^n \mapsto z^n \otimes \frac{1+g}{2} + z^{n+1} \otimes y.$$

Then $(k[z], \rho)$ is a partial comodule over H_4.
However, M contains and infinite descending chain of partial subcomodules

$$k[z] \supset k[z]^{\geq 1} \supset k[z]^{\geq 2} \supset \cdots \supset k[z]^{\geq n} \supset \cdots$$

Every element of a comodule over a coalgebra is contained in a finite-dimensional subcomodule, hence there exists no coalgebra C such that

$$\mathrm{PMod}^{H_4} \simeq \mathrm{Mod}^C.$$

Right adjoint

Theorem (E. Batista, W. H., J. Vercruysse)

The forgetful functor $U : \mathrm{PMod}^H \to \mathrm{Vect}_k$ has a right adjoint.

Right adjoint

Theorem (E. Batista, W. H., J. Vercruysse)

The forgetful functor $U : \mathrm{PMod}^H \to \mathrm{Vect}_k$ has a right adjoint.

The cofree coalgebra over H is not large enough since for a partial comodule M, the map

$$\eta : M \to M \otimes C(H) : m \mapsto (\rho^n(m))_{n \geq 0}$$

is not well-defined.

Right adjoint

Theorem (E. Batista, W. H., J. Vercruysse)

The forgetful functor $U : \mathrm{PMod}^H \to \mathrm{Vect}_k$ has a right adjoint.

The cofree coalgebra over H is not large enough since for a partial comodule M, the map

$$\eta : M \to M \otimes C(H) : m \mapsto (\rho^n(m))_{n \geq 0}$$

is not well-defined.
We will need **topological vector spaces**.

Topological vector spaces (Takeuchi)

W a vector space

- Topological basis of open subspaces $(W_\alpha)_\alpha$.
- Completion
$$\hat{W} = \varprojlim W/W_\alpha.$$
- Completed tensor product
$$W \hat{\otimes} W' = \widehat{W \otimes W'}.$$

Topological vector spaces (Takeuchi)

W a vector space

- Topological basis of open subspaces $(W_\alpha)_\alpha$.
- Completion
$$\hat{W} = \varprojlim W/W_\alpha.$$
- Completed tensor product
$$W \hat{\otimes} W' = \widehat{W \otimes W}'.$$
- Proposition :
$$\left(\prod_i W_i\right) \hat{\otimes} W' \cong \prod_i (W_i \hat{\otimes} W').$$

Topological coalgebras

Topological coalgebra C :

- Comultiplication $\Delta : C \to C\hat{\otimes}C$
- Counit $\varepsilon : C \to k$

Topological coalgebras

Topological coalgebra C :

- Comultiplication $\Delta : C \to C \hat{\otimes} C$
- Counit $\varepsilon : C \to k$

The topological cofree coalgebra on a topological vector space W is

$$\hat{C}(W) = \prod_{n \in \mathbb{N}} W^{\hat{\otimes} n}.$$

Topological coalgebras

Topological coalgebra C :

- Comultiplication $\Delta : C \to C \hat{\otimes} C$
- Counit $\varepsilon : C \to k$

The topological cofree coalgebra on a topological vector space W is

$$\hat{C}(W) = \prod_{n \in \mathbb{N}} W^{\hat{\otimes} n}.$$

Let V be a vector space and endow it with the discrete topology. Consider

$$\sigma : \ V \hat{\otimes} \hat{C}(H) \xrightarrow{V \hat{\otimes} \Delta_{\hat{C}(H)}} V \hat{\otimes} \hat{C}(H) \hat{\otimes} \hat{C}(H) \xrightarrow{V \hat{\otimes} \hat{C}(H) \hat{\otimes} p} V \hat{\otimes} \hat{C}(H) \hat{\otimes} H$$

The right adjoint

$$\sigma : \; V \hat{\otimes} \hat{C}(H) \xrightarrow{V \hat{\otimes} \Delta_{\hat{C}(H)}} V \hat{\otimes} \hat{C}(H) \hat{\otimes} \hat{C}(H) \xrightarrow{V \hat{\otimes} \hat{C}(H) \hat{\otimes} p} V \hat{\otimes} \hat{C}(H) \hat{\otimes} H$$

- Remark that $V \hat{\otimes} \hat{C}(H) = \prod_{n \geq 0} V \otimes H^{\otimes n}$.

The right adjoint

$$\sigma: \ V\hat{\otimes}\hat{C}(H) \xrightarrow{V\hat{\otimes}\Delta_{\hat{C}(H)}} V\hat{\otimes}\hat{C}(H)\hat{\otimes}\hat{C}(H) \xrightarrow{V\hat{\otimes}\hat{C}(H)\hat{\otimes}p} V\hat{\otimes}\hat{C}(H)\hat{\otimes}H$$

- Remark that $V\hat{\otimes}\hat{C}(H) = \prod_{n\geq 0} V \otimes H^{\otimes n}$.
- $R_0(V) \subseteq V\hat{\otimes}\hat{C}(H)$ maximal such that $\sigma(R_0(V)) \subseteq R_0(V) \otimes H$.

The right adjoint

$$\sigma: \ V\hat{\otimes}\hat{C}(H) \xrightarrow{V\hat{\otimes}\Delta_{\hat{C}(H)}} V\hat{\otimes}\hat{C}(H)\hat{\otimes}\hat{C}(H) \xrightarrow{V\hat{\otimes}\hat{C}(H)\hat{\otimes}p} V\hat{\otimes}\hat{C}(H)\hat{\otimes}H$$

- Remark that $V\hat{\otimes}\hat{C}(H) = \prod_{n\geq 0} V \otimes H^{\otimes n}$.
- $R_0(V) \subseteq V\hat{\otimes}\hat{C}(H)$ maximal such that $\sigma(R_0(V)) \subseteq R_0(V) \otimes H$.
- $R(V) \subseteq R_0(V)$ maximal such that $(R(V), \sigma)$ is a partial comodule.

The right adjoint

$$\sigma : \ V\hat{\otimes}\hat{C}(H) \xrightarrow{V\hat{\otimes}\Delta_{\hat{C}(H)}} V\hat{\otimes}\hat{C}(H)\hat{\otimes}\hat{C}(H) \xrightarrow{V\hat{\otimes}\hat{C}(H)\hat{\otimes}p} V\hat{\otimes}\hat{C}(H)\hat{\otimes}H$$

- Remark that $V\hat{\otimes}\hat{C}(H) = \prod_{n\geq 0} V \otimes H^{\otimes n}$.
- $R_0(V) \subseteq V\hat{\otimes}\hat{C}(H)$ maximal such that $\sigma(R_0(V)) \subseteq R_0(V) \otimes H$.
- $R(V) \subseteq R_0(V)$ maximal such that $(R(V), \sigma)$ is a partial comodule.

Unit : for a partial comodule (M, ρ)

$$\eta_M : M \to RU(M) \subseteq \prod_n M \otimes H^{\otimes n} : m \mapsto (\rho^n(m))_n.$$

Counit : for a vector space V, the restriction of

$$\mathrm{id}_V \hat{\otimes} \varepsilon_{\hat{C}(H)} : V\hat{\otimes}\hat{C}(H) \to V$$

to $UR(V)$.

Comonadicity

Theorem

The category of partial comodules over H is comonadic over Vect_k, hence it is equivalent to the Eilenberg-Moore category of the comonad $\mathbb{C} = UR$.

Apply Beck's theorem :

Comonadicity

Theorem

The category of partial comodules over H is comonadic over Vect_k, hence it is equivalent to the Eilenberg-Moore category of the comonad $\mathbb{C} = UR$.

Apply Beck's theorem :

$$\mathrm{PMod}^H \xrightarrow{\quad U \quad} \mathrm{Vect}_k$$

Comonadicity

Theorem

The category of partial comodules over H is comonadic over Vect_k, hence it is equivalent to the Eilenberg-Moore category of the comonad $\mathbb{C} = UR$.

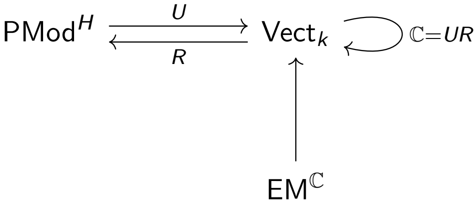
Apply Beck's theorem :

$$\mathrm{PMod}^H \underset{R}{\overset{U}{\rightleftarrows}} \mathrm{Vect}_k$$

Theorem

The category of partial comodules over H is comonadic over Vect_k, hence it is equivalent to the Eilenberg-Moore category of the comonad $\mathbb{C} = UR$.

Apply Beck's theorem :



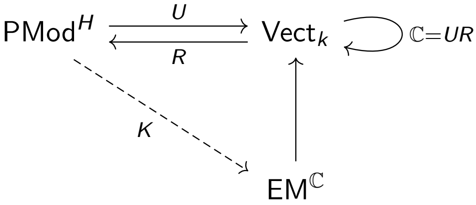
Eilenberg-Moore objects : vector space M with linear map $\rho : M \to \mathbb{C}(M)$ satisfying some conditions.

Comonadicity

Theorem

The category of partial comodules over H is comonadic over Vect_k, hence it is equivalent to the Eilenberg-Moore category of the comonad $\mathbb{C} = UR$.

Apply Beck's theorem :



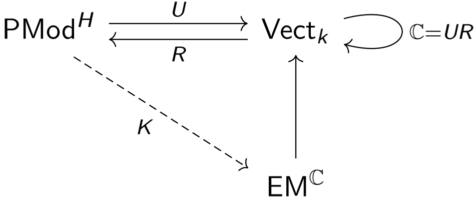
Eilenberg-Moore objects : vector space M with linear map $\rho : M \to \mathbb{C}(M)$ satisfying some conditions.

Comonadicity

Theorem

The category of partial comodules over H is comonadic over Vect_k, hence it is equivalent to the Eilenberg-Moore category of the comonad $\mathbb{C} = UR$.

Apply Beck's theorem :



Eilenberg-Moore objects : vector space M with linear map $\rho : M \to \mathbb{C}(M)$ satisfying some conditions.
$U(M)$ is a Eilenberg-Moore object

$$U(M) \xrightarrow{U(\eta_M)} \mathbb{C}U(M) = URU(M)$$

Topological partial comodules

Considering topological partial comodules $(M, \rho : M \to M \hat{\otimes} H)$, we show that

$$\overline{U} : \mathsf{TPMod}^H \to \mathsf{CHTVS}_k$$

has a right adjoint

$$\overline{R}(V) \subseteq V \hat{\otimes} \hat{C}(H)$$

and $\overline{R} \cong - \hat{\otimes} \hat{H}^{par}$.

Topological partial comodules

Considering topological partial comodules $(M, \rho : M \to M \hat{\otimes} H)$, we show that

$$\overline{U} : \mathsf{TPMod}^H \to \mathsf{CHTVS}_k$$

has a right adjoint

$$\overline{R}(V) \subseteq V \hat{\otimes} \hat{C}(H)$$

and $\overline{R} \cong - \hat{\otimes} \hat{H}^{par}$.

Conclusions

- PMod^H is equivalent to $\mathsf{EM}^{\mathbb{C}}$.
- TPMod^H is equivalent to $\mathsf{TMod}^{\hat{H}^{par}}$, the topological comodules over $\hat{H}^{par}$.
- PMod^H is equivalent to $\mathsf{DMod}^{\hat{H}^{par}}$, the discrete topological comodules over $\hat{H}^{par}$.