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Complete smooth toric varieties of Picard number 4

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Abstract

Toric varieties form a specific class of algebraic varieties equipped with a well-behaved action of an algebraic torus. They provide a useful setting for testing conjectures, as they admit a particularly explicit and combinatorial description. The fundamental theorem of toric geometry states that toric varieties correspond to fans, that is, collections of strongly convex polyhedral cones in $\mathbb{R}^n$ that are closed under taking faces and whose interiors are pairwise disjoint. Properties of the fan translate directly into geometric properties of the associated toric variety. In particular, a toric variety is *complete* if and only if the cones of the fan cover the whole space $\mathbb{R}^n$, and it is *non-singular* if and only if each cone is generated by part of a basis of the integer lattice $\mathbb{Z}^n$.

We focus here on characterizing complete non-singular toric varieties, also called *toric manifolds*.

The *Picard number* of a complete fan is the number of its 1-dimensional cones minus the dimension n; this equals the rank of the Picard group of the associated toric variety. There are two major directions of research: studying toric manifolds of fixed dimension, or studying those with fixed (small) Picard number. In dimension 2, toric manifolds are completely understood: they are obtained from either the complex projective plane or a Hirzebruch surface by a sequence of toric blow-ups. In any dimension, the unique toric

manifold of Picard number 1 is the complex projective space $\mathbb{C}P^n$. Kleinschmidt (1988) and Batyrev (1991) classified toric manifolds of Picard number 2 and 3, respectively.

In this talk, I will present a sequence of joint works with Suyoung Choi and Hyeontae Jang leading to the classification of toric manifolds of Picard number 4, using a combinatorial construction known as the *wedge operation*.